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2002 J. Phys. A: Math. Gen. 35 6743

(http://iopscience.iop.org/0305-4470/35/31/314)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 6743–6754 PII: S0305-4470(02)34090-3

# Gaussian pulses and superluminality

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Received 14 February 2002, in final form 29 May 2002
Published 26 July 2002
Online at stacks.iop.org/JPhysA/35/6743

**Abstract**
Gaussian pulses seem to be very insensitive to dispersion and therefore play an important role in the current debate about superluminal signals. We investigate the propagation of a modulated Gaussian pulse in a lossless electron plasma and show that both the centre frequency and the pulse width change significantly.

PACS numbers: 41.20.Jb, 84.40.Az

## 1. Introduction

Gaussian pulses are very popular in both theoretical and experimental work. They are smooth, providing a good time-domain model of practically achievable signals, particularly when it comes to generating short pulse durations. Related to this, the main portion of their spectrum lies within a comparatively small frequency range; hence they can be considered to be passably narrowband. Last but not least, they are easy to treat analytically. On the other hand, it is well known that both their spectral and temporal extensions are unbounded. Particularly because of its unlimited duration, a true Gaussian pulse is—strictly speaking—an inappropriate means to describe realistic signals. However, this did not affect its popularity in the past, especially in connection with the long-lasting faster-than-light debate. The reason is that the overall shape of a Gaussian-like signal seems to be relatively robust even in the presence of dispersion. If we send such a pulse through a dispersive medium, we receive on the other side of the line a signal that might be attenuated, but still resembles a Gaussian function, and, in addition, the peak of the pulse travels at a speed faster than light, because the group velocity in the medium is superluminal. This was predicted theoretically quite early [1, 2] and verified in many different experiments using media with strong absorption [3], gain-assisted anomalous dispersion [4, 5] or undersized waveguides [6, 7].

Given the resemblance of incoming and outgoing pulses, it is indeed tempting to measure the speed of signal propagation by tracing the pulse peaks. This intuitive approach caused a lot of discussion (see, e.g., [8–10] and the references therein), because the peaks are not causally related and thus carry no information [11, 12]. Consequently, many authors prefer to associate information transfer rather with abrupt changes in the signal, which in causal media travel with the speed of light [13–15]. On the other hand, it has been stated that any physically meaningful signal is necessarily bandwidth-limited [16, 17], which would prevent the existence of such wavefronts. Unfortunately, according to Fourier, this requires an infinite duration. Hence it is not possible to generate such a signal, whereas the generation of signals with limited duration is trivial and natural state-of-the-art [18–20]. It is clear that in reality, noise will affect the signal [21]. Together with the inherent low-pass character of physical devices, this puts a limit on the 'usability' of very high frequency components in a signal, but this is not tantamount to a strict bandwidth limitation.

The purpose of this paper is to investigate whether Gaussian pulses really remain unchanged when travelling through a dispersive medium. The propagation of such pulses has been studied by several authors for various media. One widely examined case is that of the classical Lorentz medium [2, 22–25]. Here, the presence of resonant absorption causes significant reshaping, especially for 'long' media. Depending on the spectral width of the input pulse, precursors or forerunners appear—even if there is no discontinuity in the signal itself—because the absorption region literally splits the spectrum of the signal. Another point of interest was the signal propagation in undersized waveguides. Some simulations also showed substantial deformations of the signal [26, 27], whereas others found no change at all [25] or only a narrowing of the pulse [28]. More recently, there have been simulations of Gaussian pulses in diffractive optical structures [29, 30] and ultracold atomic gases [31] that also yielded a shortening of the pulse.

This paper complements the work mentioned above. However, unlike these authors, who treated rather complicated media, we consider the electron plasma model, a 'friendly', well-behaved medium. This allows us to concentrate on the behaviour of the pulse itself and to avoid all disturbing effects such as excessive spectral deformation due to stop bands (such as in the case of resonant dielectrics) and problems raised by geometrical discontinuities (such as in the field simulation of waveguides).

## 2. A Gaussian signal in a dispersive medium

Not only Gaussian pulses exhibit a robust shape. Actually, all signals with a narrow spectrum have this remarkable feature, which is easily shown. Using the standard superposition approach of plane waves, we can describe the propagation of a general wave in a linear dispersive medium as a Fourier integral,

$$u(x,t) = \int_{-\infty}^{\infty} U_0(\omega)\,\mathrm{e}^{\mathrm{i}(\omega t - k(\omega)x)}\,\mathrm{d}\omega \tag{1}$$

with $U_0(\omega)$ as the spectrum of the original signal at $x = 0$. In the case of a modulated signal $u_0(t) = u_e(t)\,\mathrm{e}^{\mathrm{i}\omega_c t}$ with an envelope $u_e(t)$ and a carrier frequency $\omega_c$, we know that its spectrum is that of the envelope, but shifted by $\omega_c$, hence $U_0(\omega) = U_e(\omega - \omega_c)$. If we now expand the dispersion relation $k(\omega)$ in a Taylor series about the carrier frequency and neglect the terms of higher than linear order, such that $k(\omega) \approx k_c + (\omega - \omega_c)/v_g$, we finally find for the wave

$$u(x,t) = \mathrm{e}^{\mathrm{i}(\omega_c t - k_c x)} \int_{-\infty}^{\infty} U_e(\omega - \omega_c) \exp\left(\mathrm{i}\left((\omega - \omega_c)t - \frac{x}{v_g}k(\omega - \omega_c)\right)\right) \mathrm{d}\omega \tag{2}$$

which shows that the envelope preserves its original shape and moves with the group velocity $v_g = \mathrm{d}\omega/\mathrm{d}k|_{\omega_c}$. Only the carrier proceeds with the phase velocity $v_p = \omega_c/k_c$ underneath the envelope. This finding is not spectacular at all and well known [13]. However, it is the anchor point for the widespread belief that the group velocity adequately describes the motion of a wave and the pulse shape remains unchanged if the spectrum of the signal is sufficiently narrow [32–35]. Still, the linearization of the dispersion relation is only a crude approximation, and the wave packet does not really move with $v_g$ if higher-order terms of the expansion are taken into account [36, 37]. Another natural consequence is that the signal will lose its original shape [38], be it Gaussian or not.

The dispersive medium of choice for our investigation is the lossless electron plasma, which has the advantage of being well-behaved, without any resonance effects interfering with the perception of the primary effects of dispersion. The dispersion relation is given by [39]

$$k(\omega) = \frac{\omega_p}{c}\sqrt{\left(\frac{\omega}{\omega_p}\right)^2 - 1} \tag{3}$$

where $\omega_p$ is the plasma or cut-off frequency denoting the lower end of the passband. This dispersion relation is the same as that of a hollow waveguide. In the plasma, however, we may simply describe the signals as TEM waves, without having to care for geometric boundary conditions. This is exactly true if the plasma is completely unbounded, i.e. if there are no boundaries perpendicular to the propagation direction. But it is by no means just a convenient simplification of reality. It also gives correct results for the practically relevant case of transmission lines such as twisted pair or coaxial cables, provided the dielectric is appropriately dispersive.

For practical use, care must be taken with the evaluation of the square root in (3). In order to obtain correct results, we need to distinguish three frequency ranges [20]

$$k(\omega) = \begin{cases} -\frac{1}{c}\sqrt{\omega^2 - \omega_p^2} & \text{if} \quad \omega < -\omega_p \\ -\frac{\mathrm{i}}{c}\sqrt{\omega_p^2 - \omega^2} & \text{if} \quad |\omega| \leqslant \omega_p \\ \frac{1}{c}\sqrt{\omega^2 - \omega_p^2} & \text{if} \quad \omega > \omega_p. \end{cases} \tag{4}$$

For our analysis, the evanescent range $|\omega| \leqslant \omega_p$ is the most interesting. Below cutoff, the wavenumber becomes imaginary, and the frequency components of the signal experience no phase shift, but only attenuation in the $x$-direction. Therefore we have no wave propagation in the original sense, and the signal seems to be the same—except for an exponential decay—at any point along the line. Still, this does not mean that the transfer function of the medium is frequency-invariant and thus distortionless [20]. In the following, we also presume that the medium is infinitely long or at least properly terminated, so as to avoid reflections.

The signal we apply to this dispersive medium is characterized by a Gaussian envelope modulating an arbitrary carrier oscillation. The original signal at $x = 0$ is given by

$$u(0, t) = u_0\,\mathrm{e}^{-(\frac{t-t_0}{\sigma})^2}\,\mathrm{Re}\,\mathrm{e}^{\mathrm{i}\omega_0 t} \tag{5}$$

where $t_0$ denotes the temporal position of the peak and $\sigma$ is the standard deviation (temporal width) of the envelope. The frequency $\omega_0$ is the carrier frequency. Owing to the linearity of the system and for the sake of simplicity, we may use complex notation and take the real part of the expressions when necessary. The spectrum $U_0(\omega)$ of the initial pulse can easily be calculated, which gives

$$U_0(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} u(0, t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t = \frac{u_0\sigma}{2\sqrt{\pi}}\,\mathrm{e}^{-(\frac{\sigma(\omega-\omega_0)}{2})^2}\,\mathrm{e}^{-\mathrm{i}t_0(\omega-\omega_0)}. \tag{6}$$

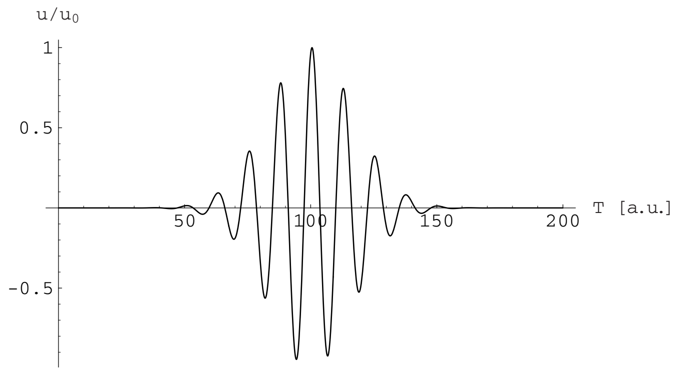


**Figure 1.** Initial Gaussian pulse (5) for the parameter values $T_0 = 100$, $\Omega = 0.5$ and $\Sigma = 24$. All quantities are dimensionless according to definitions (8) and (9). For a cut-off frequency $\omega_p = 6 \times 10^{10}\ \mathrm{s}^{-1}$, the pulse width is about 1.7 ns.

To allow for an efficient numerical computation of the Fourier integral (1), it is reasonable to dispense with the various constants and to introduce a normalized integration variable

$$\eta = \frac{\omega}{\omega_p} \tag{7}$$

as well as normalized parameters

$$T = \omega_p t \qquad T_0 = \omega_p t_0 \qquad X = \frac{\omega_p x}{c} \tag{8}$$

and

$$\Sigma = \omega_p \sigma \qquad \Omega = \frac{\omega_0}{\omega_p}. \tag{9}$$

With these new variables, we can finally write the signal at any position along the line as

$$\frac{u(X,T)}{u_0} = \frac{\Sigma}{2\sqrt{\pi}} \mathrm{Re} \int_{-\infty}^{\infty} \mathrm{e}^{-\left(\frac{\Sigma(\eta-\Omega)}{2}\right)^2} \mathrm{e}^{\mathrm{i}(\eta(T-T_0)+\eta\Omega-K(\eta)X)}\, \mathrm{d}\eta \tag{10}$$

with the normalized dispersion relation $K(\eta) = \sqrt{\eta^2 - 1}$. The actual expressions for $K(\eta)$ in the individual frequency ranges must be chosen according to (4).

## 3. Numerical evaluation

The expression for the signal (10) is valid in both the passband and the evanescent region. We can thus compute the signal inside the plasma for a given pulse with varying carrier frequencies. As we want to examine evanescent signals, we select the carrier frequency from below cutoff. Figure 1 shows such a signal. If the cut-off frequency of the plasma is assumed to be $\omega_p = 60 \times 10^9\ \mathrm{s}^{-1}$ (a value roughly corresponding to the microwave experiments described in [6]), then the pulse width is about 1.7 ns, which is fairly short, but in the range of pulse lengths actually used in communication technology. Accordingly, the spectrum is quite wide, as shown in figure 2. Still the value of $|U_0(\omega)|$ at cutoff is only $1.57 \times 10^{-15}$ and so the spectral components outside the evanescent region seem negligible.

The numerical evaluation of the Fourier integral (10) is comparatively straightforward. Although the integrand is irregularly oscillating, which is known to cause problems with numerical quadrature [40], a truncation of the integration interval is possible, and we can

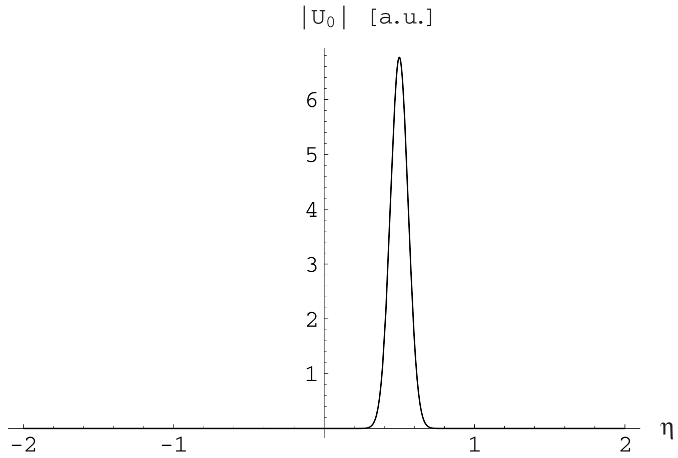


**Figure 2.** Spectrum (6) of the evanescent Gaussian pulse shown in figure 1. The normalized frequency value $|\eta| = 1$ marks the cut-off frequency of the medium.

estimate the truncation error. If we neglect the oscillating factor and regard only the envelope of the spectrum, we find an upper bound for the error

$$\varepsilon = \frac{\Sigma}{\sqrt{\pi}} \int_{\Omega+\Delta}^{\infty} \mathrm{e}^{-\left(\frac{\Sigma(\eta-\Omega)}{2}\right)^2} \, \mathrm{d}\eta = 1 - \mathrm{erf}\left(\frac{\Sigma}{2}\Delta\right) \tag{11}$$

for a symmetric integration interval $[\Omega - \Delta, \Omega + \Delta]$. Actually, the error will be significantly smaller for two reasons: firstly, a part of the truncated tails will most likely lie in the evanescent range $|\eta| < 1$, where an additional attenuation occurs. Secondly and more important, the oscillatory behaviour inherently limits the error. Normally, the error is determined by the integral of the half-period next to the truncation point. Unfortunately, when the argument function of the oscillation factor is hyperbolic as in the present case, this rule of thumb is not sufficient to derive a reliable upper bound without a detailed analysis of the argument function [40]. However, the Gaussian envelope of the integrand allows us to define a conservative bound that can be made arbitrarily small. Quadrature itself is best carried out with an algorithm suitable for oscillatory integrands, such as the double-exponential rule.

The space-time contour plot in figure 3 shows the numerical evaluation of the signal amplitude (10) for various values of the parameters $X$ and $T$. As we are investigating a one-dimensional problem, we can combine the temporal and spatial evaluation of the signal in a two-dimensional diagram. Hence, the plot can be read either as a sequence of snapshots (for constant $T$) of the signal along the line or as a collection of pulse recordings for every point in the medium (with $X$ held constant). In this way, the figure depicts the wave as it penetrates the medium. To allow for a better recognition of the details, the plot range of the signal value is restricted to a very small amplitude range around zero. All values outside this range are clipped and coloured black and white. So, what we actually see are the zero crossings of the signal. The small plot range is also the reason why the initial signal at $X = 0$, which is identical to that shown in figure 1, exhibits more oscillations than figure 1 would lead one to expect at first sight.

Figure 3 reveals a few remarkable features of the Gaussian signal. First of all, we notice that the bulk of the signal energy is concentrated within a roughly parabola-shaped region. This part corresponds to the initial Gaussian pulse. Note that although it looks as if the pulse became shorter with increasing $X$, this is not the case. It is only the exponential attenuation

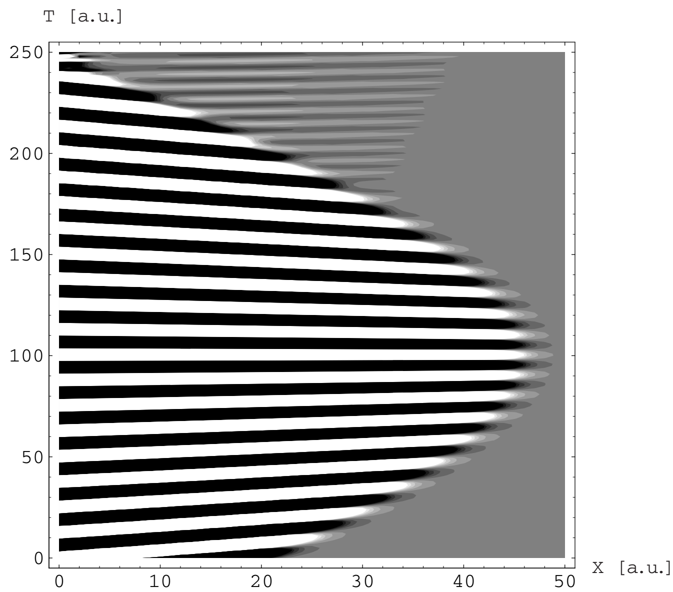


**Figure 3.** Evolution of the initially Gaussian signal (10) in the medium. The signal range covered by the contour plot is $u/u_0 \in [-4 \times 10^{-17}, 4 \times 10^{-17}]$. The truncation of the integration interval was chosen so as to limit the error (11) to $\varepsilon = 8.4 \times 10^{-21}$. The time and space coordinates are dimensionless according to (8).

combined with the limited plotting range that creates this deceptive impression. The overall shape of the pulse remains more or less the same, albeit with reduced amplitude. The figure also suggests that the peak of the pulse envelope $|u/u_0|$ is always detected at the same time $T = T_0$, according to the postulation that evanescence causes only decay and no phase shift. Indeed, a closer examination reveals that the pulse peak is slightly retarded deeper inside the medium. At $X = 50$, for instance, the peak turns up at $T = 100.251$. It is noteworthy that this delay increases progressively with $X$, until the pulse becomes too small and numerical errors prevail.

Besides the main pulse, however, we also recognize a small fraction running away down the line when the major part of the signal is already diminishing. The high oscillation frequency (twice the carrier frequency) of this low but very broad pulse shows that it consists of the spectral components just above cutoff. Obviously, we cannot neglect the spectral components beyond the cut-off frequency, although they are 15 orders of magnitude smaller than the peak. To verify that this part of the signal is not due to numerical artefacts, we can roughly calculate the frequency that dominates this part of the signal by measuring the slope $\Delta X/\Delta T$ parallel to the peaks and troughs visible in the graph. They give an estimate for the phase velocity. Other approaches such as the commonly used stationary phase method are not applicable because the rapidly decaying envelope of the spectrum overshadows the influence of the phase of the oscillating integrand. Using the definition of the dispersion relation (3), we readily find the normalized expression for the phase velocity,

$$\frac{v_p}{c} = \sqrt{\frac{\Omega^2}{\Omega^2 - 1}}. \tag{12}$$

With this equation, which directly corresponds to the appropriately measured slope $\Delta X/\Delta T$, we finally obtain an interval for the carrier frequency, $\Omega \in [1, 1.02]$. The left bound of the

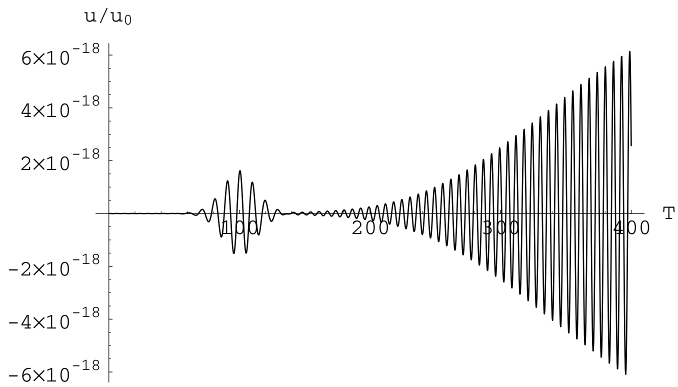


**Figure 4.** The original signal from figure 1 at a normalized distance $X = 50$ from the signal source. The dominating part consists of spectral components above cutoff.

interval is obtained from the left end of the propagating wave close to $X = 0$, where the tangents to the ripples are horizontal. The same consideration applied to the right end of the wave gives the upper limit of the interval.

Figure 4 depicts the very interesting shape of the signal at $X = 50$, a substantial distance from the signal source. This corresponds exactly to the right edge of figure 3, where the signal was already too small to be displayed properly. However, the Gaussian pulse is still present. But the small wavepacket around $T = 100$ is only a faint copy of the original pulse of figure 1, and the propagating part dominates the scene. The envelope of this part reaches its maximum at $T \approx 600$ (at an amplitude value of about $8 \times 10^{-18}$) and slowly decays afterwards. We recall that the numerical evaluation of the signal was done over a finite frequency interval, which comes down to treating the signal as strictly bandwidth-limited. Yet it is important to notice that the ripples are not acausal precursors originating from the truncation of the integration range, but genuine signal components. According to (11), the quadrature error is smaller than $8.4 \times 10^{-21}$, which is well below the amplitude range shown in figure 4. One must, of course, not overlook the fact that the amplitude of the propagating part (which is nearly independent of the distance from the source) is 17 orders of magnitude smaller than the initial main pulse. Therefore it is likely to be missed in an experiment.

## 4. Particular properties of the signal

The time-domain snapshots presented before already suggest that the Gaussian pulse does not retain its original shape while travelling through the medium. We shall examine this effect in more detail now by considering two basic properties that characterize the signal: the centre frequency of the spectrum and the width of the envelope. The extent to which they change can serve as a criterion to judge whether or not the assumption that the signal remains unaltered is justified.

As for the centre frequency (which is effectively the carrier frequency), it is particularly noteworthy that it increases gradually with $X$. This is manifested in figure 3 by the narrowing of the black and white stripes, and is nothing but a filter effect. Since the dispersion in the plasma causes higher-frequency components to be less attenuated than lower ones, the centre

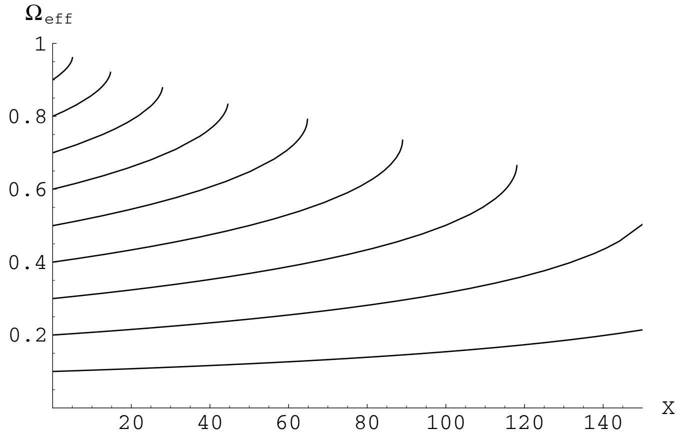


**Figure 5.** Effective normalized carrier frequency $\Omega_{\mathrm{eff}}$ for the Gaussian signal (10) with varying initial carrier frequency $\Omega$ and $\Sigma = 24$ depending on the normalized position along the medium.

frequency of the spectrum is slightly raised. From the filtered spectrum,

$$|U(\eta)| = \frac{u_0 \Sigma}{2\sqrt{\pi}} \, \mathrm{e}^{-(\frac{\Sigma(\eta-\Omega)}{2})^2 - X\sqrt{1-\eta^2}} \tag{13}$$

we can easily deduce the effective carrier frequency, which corresponds to the maximum of the spectrum in the evanescent region. Figure 5 shows the result for the example considered, but for varying initial carrier frequency $\Omega$. With growing distance from the signal source, the centre frequency increases nearly linearly at first, and then progressively. At a certain point down the line, the curve suddenly terminates. This is where the spectral contribution of the formerly Gaussian signal vanishes entirely. The criterion used here is the existence of a local spectral maximum of $|U(\eta)|$ for $\eta < 1$. As the pulse moves away from the source, its originally Gaussian spectrum (figure 2) is—apart from the frequency shift—attenuated exponentially by the frequency-dependent factor $\exp\left(-X\sqrt{1-\eta^2}\right)$. This attenuation is less severe, close to $\eta = 1$, irrespective of the value of $X$, whereas the spectrum of the original signal diminishes rapidly. In the spectrum (13) this is manifested by a steep rise of $|U(\eta)|$ for $\eta \to 1$. As $X$ increases, the peak of the Gaussian spectral part will finally be swallowed by the less-attenuated components (i.e. it degenerates to a saddle point in $|U(\eta)|$), and the local spectral maximum will cease to exist. From a frequency-domain point of view, this marks the ultimate disappearance of the Gaussian pulse. Note that the Gaussian-like pulse shape in the time domain hinges on a clear separation of the main Gaussian part of the spectrum and the tail favoured by the transfer function of the medium. In figure 4 this is still the case. For larger values of $X$, the shape of the pulse envelope becomes more and more distorted, until it is no longer recognizable.

It is not surprising that the frequency-shift effect in figure 5 is stronger for high carrier frequencies. The transfer function of the medium at a given point $X$ is essentially proportional to $\exp\left(-\sqrt{1-\eta^2}\right)$, which has an increasing derivative for $\eta \to 1$. Therefore the favouring of the high-frequency parts of the signal spectrum is more pronounced if $\Omega$ is high. Conversely, the effect is not present at all if $\Omega = 0$, i.e., if there is no modulation and only the baseband signal is used. In this case, we have only the signal's envelope without carrier, and the centre frequency remains the same, independent of the position along the line. The reason for this behaviour is obviously the symmetrical shape of the dispersion relation.

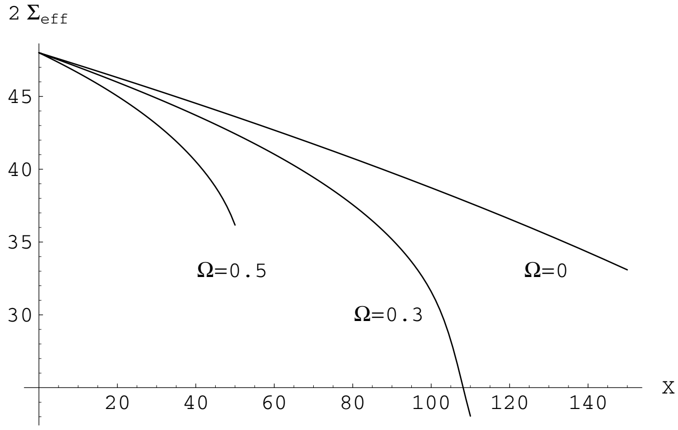


**Figure 6.** Effective normalized standard deviation $\Sigma_{\text{eff}}$ for the envelope $|u(X,T)|$ of the Gaussian signal (10) depending on the normalized position along the medium for varying initial carrier frequency $\Omega$ and $\Sigma = 24$.

The second critical property of the Gaussian pulse is its width. It is even more important than the centre frequency of the spectrum since the latter is normally introduced by modulation to adapt the signal to the communication channel. However, this is done *after* the baseband signal containing the actual information to be transmitted is generated. Therefore, and also with respect to the introductory remarks concerning equation (2), the width as a property of the envelope is more closely connected to the 'signal' itself than the carrier.

In order to obtain the actual width of the pulse, we have to numerically evaluate the signal (10) for a given point $X$ and find the maximum of the envelope, $|u(X,T)|_{\max}$. Next, we have to find the two solutions of the equation

$$\{T_{\min}, T_{\max}\} : |u(X,T)| = \frac{1}{\mathrm{e}}|u(X,T)|_{\max} \tag{14}$$

and can define the effective standard deviation of the pulse (presuming that it is still Gaussian-like),

$$\Sigma_{\text{eff}} = \frac{T_{\max} - T_{\min}}{2}. \tag{15}$$

Figure 6 shows the results for the same parameters as before, again with varying carrier frequency $\Omega$. We clearly see that the width does change as the pulse moves through the medium. Again, the effect is more marked for high carrier frequencies and progressively increases with $X$. Qualitatively, these results are in line with the findings of [28]. At some point, it is no longer possible to reliably solve equation (14), because the signal values are too small or the shape of the pulse ceases to be Gaussian-like, respectively. What is slightly astonishing is the fact that the width diminishes also for pure baseband signals. The impact is smaller than that for modulated signals, but it is noticeable. Even worse, the derivative of $\Sigma_{\text{eff}}(X)$ at $X = 0$ does not vanish. Hence, we cannot even argue that the envelope of the pulse remains roughly unchanged in the vicinity of the entrance to the medium.

## 5. Conclusion

The investigation of Gaussian signals in the evanescent regime of an electron plasma has brought some remarkable, yet not at all unexpected findings to light. Evidently, the peak of the pulse's envelope traverses a short distance of the medium in almost no time or at least with only a short delay. At first sight, this could be seen as some sort of 'local' superluminality, as discussed in [41]. However, although it is tempting to identify the trajectory of the pulse peak with the signal, the points of the trajectory are not causally related. Apart from this general problem, some other basic properties of Gaussian pulses that are often praised also do not withstand a closer examination. In particular, the width of the signal does not, even in an approximate sense, remain constant—not even in this well-behaved model we considered. Furthermore, as the spectrum is not truly limited (despite the exponential decay on either side of the carrier frequency), propagating signal parts still exist. Eventually, they completely overshadow the evanescent components of the pulse. This leads to a severe alteration of the signal shape, and the assumption that the pulse remains Gaussian no longer holds. In addition, the highpass filter properties of the medium cause a frequency shift of the signal spectrum. Surely one can argue that provided the pulse is sufficiently wide, the propagating componenents can be made arbitrarily small. But just because we may not be able to find them in the noise of an experiment, we cannot conclude that they are not there.

How can the findings of the preceding sections be related to the experimental results, where allegedly no changes of the Gaussian-like signals were found? First of all, our model can be applied only to the waveguide experiments because of the equivalent dispersion relation. For the results reported in [6], we have the parameter values $\Omega = 0.917$ (for a carrier of 8.7 GHz and the cut-off frequency 9.47 GHz), $\Sigma = 76$ (which roughly corresponds to the described spectral width of 0.5 GHz) and $X = 20$ (for the longest barrier considered in the experiments). With these values, we obtain a reduction of the effective standard variation $\Sigma_{\rm eff}$ by 10.2%. This may seem high, but given the strong attenuation of $5.75 \times 10^{-4}$ or 65 dB together with the inevitable measurement noise, this change of the overall signal shape may easily be overlooked.

The situation is different for the simulation results presented in [25]. These authors used very long pulses (compared to the cut-off frequency of the waveguide), which give the parameters $\Omega = 0.967$ (carrier at 14.5 GHz, cutoff at 15 GHz), $\Sigma = 942$ (for a temporal width of about $\sigma = 10$ ns according to the signal shape shown in the reference) and $X = 10$ (for a waveguide length of 32.96 mm). Here we obtain a reduction of $\Sigma_{\rm eff}$ by only 0.07% and an attenuation of $7.84 \times 10^{-2}$ or 22 dB. In this case, the change of shape is indeed negligible.

For the sake of completeness, it should be noted that there have also been critical reviews of other experiments where no deformation of the signal shape was observed (see, e.g., [42] for the experiments with strongly absorbing media [3]). Even for the recent experiment with transparent anomalous dispersion between two resonance regions [5], a theoretical analysis revealed that there must have been some distortion [38]. What is needed for true conservation of the signal envelope can be seen from equation (2): a dispersion relation $k(\omega) = k_c + (\omega - \omega_c)/v_g$ that is linear over the entire frequency range of interest, or, in optical terms, a refractive index of the medium that is hyperbolic according to $n(\omega) = ck(\omega)/\omega$ (and not linear as erroneously stated in [5]).

The basic effects of dispersion investigated here (pulse width distortion and frequency shift) are quite universal and independent of the concrete medium. Likewise, they are not limited to the simplified case of one-dimensional pulses, but will also appear in three-dimensional pulses, where diffraction causes additional distortion leading to superluminal pulse propagation even in free space [43]. Still, the resulting wave functions can, to a certain extent, be formally described and treated like one-dimensional waves in a dispersive medium [44]. However, the superluminal phenomena found in such beams are often interference effects, and the one-dimensional expressions may lead to false interpretations [45].

A completely different question is whether Gaussian-like signals are suitable to transfer information superluminally. It is clear that true Gaussian pulses are only a convenient model and not realizable because of their unlimited duration [46]. An illustrative instance is figure 3, where at $T = 0$ obvious carrier oscillations can be found at the signal source, even though in figure 1 the medium *seems* to be quiet at that time. On the other hand, also time-limited, technically relevant signals can be smooth, like raised cosine pulses [20] which are slew-rate limited and also similar to Gaussian functions. The distinct start and end points of such pulses inevitably create higher frequency signal components only moving with $c$. It has been argued that superluminal information transfer is theoretically possible if these wavefronts can be kept small, but limited by the inverse bandwidth of the signal and therefore practically useless [19]. The problem with all experiments and simulations investigating superluminal signals is that they are based on time-of-flight measurements comparing pulses traversing a dispersive medium with reference signals moving in free air. From the point of information transfer, however, this is unjustified, because the transfer starts as soon as the pulse is generated by the source and not only when the peak appears. When talking about the possibility of superluminal signals, one should also take this additional delay into account, which is much more significant than the relative advancement of pulse peaks or other points of the signal envelope that are not causally related anyway.

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